Identify the vertex, the axis of symmetry, the maximum or minimum value, and the range of each parabola.

1. \( y = x^2 - 4x + 1 \)
2. \( y = -x^2 + 2x + 3 \)

3. \( y = -x^2 - 6x - 10 \)
4. \( y = 3x^2 + 18x + 32 \)

5. \( y = 2x^2 + 3x - 5 \)
6. \( y = -3x^2 + 4x \)

Graph each function.

7. \( y = x^2 + 2x - 5 \)
8. \( y = -x^2 + 3x + 1 \)

9. \( y = 2x^2 + 4x - 4 \)
10. \( y = -\frac{1}{2}x^2 - 3x + 3 \)

11. \( y = 3x^2 - 8x \)
12. \( y = -3x^2 + 18x - 27 \)

Write each function in vertex form.

13. \( y = x^2 - 8x + 19 \)
14. \( y = x^2 - 2x - 6 \)

15. \( y = x^2 + 3x \)
16. \( y = 2x^2 + x \)

17. \( y = 2x^2 - 12x + 11 \)
18. \( y = -2x^2 - 4x + 6 \)
Factor each expression.

1. \( x^2 + 11x + 28 \)  
2. \( x^2 + 11x + 24 \)
3. \( s^2 + 15s + 42 \)  
4. \( x^2 - 10x + 21 \)
5. \( y^2 - 8y + 15 \)  
6. \( x^2 - 12x + 32 \)
7. \( -x^2 + 9x - 18 \)  
8. \( -w^2 + 12w - 35 \)
9. \( -t^2 - 3t + 54 \)  
10. \( x^2 - 7x - 60 \)

Find the GCF of each expression. Then factor the expression.

11. \( 6x^2 - 9 \)  
12. \( 16m^2 + 8m \)
13. \( 2a^2 + 22a + 60 \)  
14. \( 5x^2 + 25x - 70 \)
15. \( \frac{1}{3}x^2 + \frac{1}{3}x - 4 \)  
16. \( -7x^2 + 7x + 14 \)

Factor each expression.

17. \( 5x^2 - 17x + 6 \)  
18. \( 3x^2 + 10x + 8 \)
19. \( 2b^2 - 9b - 5 \)  
20. \( z^2 + 12z + 36 \)
21. \( 9x^2 - 6x + 1 \)  
22. \( 4k^2 + 12k + 9 \)
23. \( n^2 - 49 \)  
24. \( 2t^2 - 50 \)

25. The area of a rectangular field is \( x^2 - x - 72 \text{ m}^2 \). The length of the field is \( x + 8 \text{ m} \). What is the width of the field in meters?
5-1 Practice
Polynomial Functions

Write each polynomial in standard form. Then classify it by degree and by number of terms.

1. \(4x + x + 2\)  
2. \(-3 + 3x - 3x\)  
3. \(6x^4 - 1\)

4. \(1 - 2x + 5x^4\)  
5. \(5m^2 - 3m^2\)  
6. \(x^2 + 3x - 4x^3\)

7. \(-1 + 2x^2\)  
8. \(5m^2 - 3m^3\)  
9. \(5x - 7x^2\)

10. \(2 + 3x^3 - 2\)  
11. \(6 - 2x^3 - 4 + x^3\)  
12. \(6x - 7x\)

13. \(a^3(a^2 + a + 1)\)  
14. \(x(x + 5) - 5(x + 5)\)  
15. \(p(p - 5) + 6\)

16. \((3c^2)^2\)  
17. \(-3 - b\)  
18. \(n(2x - 1)\)

19. \(\frac{2}{3} + s^2\)  
20. \(\frac{2x^4 + 4x - 5}{4}\)  
21. \(\frac{3 - x^5}{3}\)

Determine the end behavior of the graph of each polynomial function.

22. \(y = 3x^4 + 6x^3 - x^2 + 12\)  
23. \(y = 50 - 3x^3 + 5x^2\)  
24. \(y = -x + x^2 + 2\)

25. \(y = 4x^2 + 9 - 5x^4 - x^3\)  
26. \(y = 12x^4 - x + 3x^7 - 1\)  
27. \(y = 2x^5 + x^2 - 4\)

28. \(y = 5 + 2x + 7x^2 - 5x^3\)  
29. \(y = 20 - 5x^6 + 3x - 11x^3\)  
30. \(y = 6x + 25 + 4x^4 - x^2\)

Describe the shape of the graph of each cubic function by determining the end behavior and number of turning points.

31. \(y = x^3 + 4x\)  
32. \(y = -2x^3 + 3x - 1\)  
33. \(y = 5x^3 + 6x^2\)

Determine the degree of the polynomial function with the given data.

34. \[
\begin{array}{c|c}
-2 & -16 \\
-1 & 1 \\
0 & 4 \\
1 & 5 \\
2 & 16 \\
\end{array}
\]

35. \[
\begin{array}{c|c}
-2 & 52 \\
-1 & 6 \\
0 & 2 \\
1 & 4 \\
2 & 48 \\
\end{array}
\]
5-4 Practice
Dividing Polynomials

Divide using long division. Check your answers.

1. \((x^3 - 3x^2 - 10) \div (x + 3)\)
2. \((2x^2 + x - 7) \div (x - 5)\)
3. \((x^3 + 5x^2 - 3x - 1) \div (x - 1)\)
4. \((3x^3 - x^2 - 7x + 6) \div (x + 2)\)
5. \((x^2 - 3x + 1) \div (x - 4)\)
6. \((x^3 - 4x^2 + 3x + 2) \div (x + 2)\)

Determine whether each binomial is a factor of \(x^3 + 3x^2 - 10x - 24\).

7. \(x + 4\)
8. \(x - 3\)
9. \(x + 6\)
10. \(x + 2\)

Divide using synthetic division.

11. \((x^3 - 8x^2 + 17x - 10) \div (x - 5)\)
12. \((x^3 + 5x^2 - x - 9) \div (x + 2)\)
13. \((-2x^3 + 15x^2 - 22x - 15) \div (x - 3)\)
14. \((x^3 + 7x^2 + 15x + 9) \div (x + 1)\)
15. \((x^3 + 2x^2 + 5x + 12) \div (x + 3)\)
16. \((x^3 - 5x^2 - 7x + 25) \div (x - 5)\)
17. \((x^4 - x^3 + x^2 - x + 1) \div (x - 1)\)
18. \((2x^4 + 7x^3 - 11x^2 + 21x + 5) \div (x + 5)\)
19. \((x^4 - 5x^3 + 5x^2 + 7x - 12) \div (x - 4)\)
20. \((2x^4 + 23x^3 + 50x^2 - 125x - 500) \div (x + 4)\)

Use synthetic division and the given factor to completely factor each polynomial function.

21. \(y = x^3 + 3x^2 - 13x - 15; (x + 5)\)
22. \(y = x^3 - 3x^2 - 10x + 24; (x - 2)\)
23. \(y = x^3 + x^2 - 10x + 8; (x - 1)\)
24. \(y = x^3 + 4x^2 - 9x - 36; (x + 3)\)

25. The expression \(V(x) = x^3 - 13x + 12\) represents the volume of a rectangular safe in cubic feet. The length of the safe is \(x + 4\). What linear expressions with integer coefficients could represent the other dimensions of the safe? Assume that the height is greater than the width.

Use synthetic division and the Remainder Theorem to find \(P(a)\).

26. \(P(x) = 3x^3 - 4x^2 - 5x + 1; a = 2\)
27. \(P(x) = x^3 + 7x^2 + 12x - 3; a = -5\)
28. \(P(x) = x^3 + 6x^2 + 10x + 3; a = -3\)
29. \(P(x) = 2x^4 - 9x^3 + 7x^2 - 5x + 11; a = 4\)
Determine the cubic function that is obtained from the parent function $y = x^3$ after each sequence of transformations.

1. a reflection in the $x$-axis; 
   a vertical translation 3 units down; 
   and a horizontal translation 2 units right

2. a vertical stretch by a factor of 4; 
   a reflection in the $x$-axis; 
   and a horizontal translation $\frac{1}{2}$ unit left

3. a vertical stretch by a factor of $\frac{1}{3}$; 
   a reflection in the $y$-axis; 
   and a vertical translation 6 units up

4. a vertical stretch by a factor of 3; 
   a reflection in the $x$-axis; 
   a vertical translation 2 units down; 
   and a horizontal translation 2 units left

Find all the real zeros of each function.

5. $y = 2(x + 1)^3 - 3$

6. $y = -3(x - 2)^3 + 24$

7. $y = -\frac{1}{2}(x + 4)^3 - 1$

8. $y = 8(-x - 2)^3 + 5$

9. $y = -(x + 5)^3 + 1$

10. $y = 4(x - 6)^3 - 2$

Find a quartic function with the given $x$-values as its only real zeros.

11. $x = 2$ and $x = 8$

12. $x = 3$ and $x = -1$

13. $x = 1$ and $x = 3$

14. $x = -2$ and $x = 6$

15. $x = 5$ and $x = -2$

16. $x = -1$ and $x = 2$

17. $x = -3$ and $x = -5$

18. $x = -4$ and $x = 4$

19. **Physics** If you stretch a spring to 5 ft, it has 310 ft-lb of potential energy ($PE$). Potential energy varies directly as the square of the stretched length ($l$). The potential energy can be represented by the formula $PE = \frac{1}{2}k l^2$, where $k$ is the spring constant.
   
   a. What is the value of the spring constant for this spring?

   b. How many ft-lbs of $PE$ would an 8 ft length of spring have?
8-3 Practice

Rational Functions and Their Graphs

Find the domain, points of discontinuity, and x- and y-intercepts of each rational function. Determine whether the discontinuities are removable or nonremovable.

1. \( y = \frac{(x - 4)(x + 3)}{x + 3} \)

2. \( y = \frac{(x - 3)(x + 1)}{x - 2} \)

3. \( y = \frac{2}{x + 1} \)

4. \( y = \frac{4x}{x^4 + 16} \)

Find the vertical asymptotes and holes for the graph of each rational function.

5. \( y = \frac{5 - x}{x^2 - 1} \)

6. \( y = \frac{x^2 - 2}{x + 2} \)

7. \( y = \frac{x}{x(x - 1)} \)

8. \( y = \frac{x + 3}{x^2 - 9} \)

9. \( y = \frac{x - 2}{(x + 2)(x - 2)} \)

10. \( y = \frac{x^2 - 4}{x^2 + 4} \)

11. \( y = \frac{x^2 - 25}{x - 4} \)

12. \( y = \frac{(x - 2)(2x + 3)}{(5x + 4)(x - 3)} \)

Find the horizontal asymptote of the graph of each rational function.

13. \( y = \frac{2}{x - 6} \)

14. \( y = \frac{x + 2}{x - 4} \)

15. \( y = \frac{2x^2 - 3}{x^2 - 6} \)

16. \( y = \frac{3x - 12}{x^2 - 2} \)

Sketch the graph of each rational function.

17. \( y = \frac{3}{x - 2} \)

18. \( y = \frac{3}{(x - 2)(x + 2)} \)

19. \( y = \frac{x}{x^2 + 4} \)

20. \( y = \frac{x + 2}{x - 1} \)
Find the least common multiple of each pair of polynomials.

1. $3x(x + 2)$ and $6x(2x - 3)$
2. $2x^2 - 8x + 8$ and $3x^2 + 27x - 30$
3. $4x^2 + 12x + 9$ and $4x^2 - 9$
4. $2x^2 - 18$ and $5x^3 + 30x^2 + 45x$

Simplify each sum or difference. State any restrictions on the variables.

5. $\frac{x^2}{5} + \frac{x^2}{5}$
6. $\frac{5y - 4}{y^2 - 5} + \frac{3y + 1}{y^2 - 5}$
7. $\frac{2y + 1}{3y} + \frac{5y + 4}{3y}$
8. $\frac{12}{xy^3} - \frac{9}{xy^3}$
9. $\frac{2}{n + 4} - \frac{n^2}{n^2 - 16}$
10. $\frac{3}{8x^3y^3} - \frac{1}{4xy}$
11. $\frac{6}{5x^2y} + \frac{5}{10xy^2}$
12. $\frac{x + 2}{x^2 + 4x + 4} + \frac{2}{x + 2}$
13. $\frac{4}{x^2 - 25} + \frac{6}{x^2 + 6x + 5}$
14. $\frac{y}{4y + 8} - \frac{1}{y^2 + 2y}$

Simplify each complex fraction.

15. $\frac{\frac{2x}{3}}{\frac{3}{y}}$
16. $\frac{1 + \frac{2}{x}}{4 - \frac{6}{x}}$
17. $\frac{\frac{1}{x - 3}}{2 + \frac{1}{x}}$
18. $\frac{\frac{x + 1}{5}}{\frac{x - 1}{x}}$
19. $\frac{\frac{x^2 - 1}{3}}{\frac{x + 1}{x}}$
20. $\frac{1 + \frac{3}{4}}{9}$
21. $\frac{\frac{2x + 6}{1}}{\frac{1}{y}}$
22. $\frac{\frac{x + 3}{x - 3}}{\frac{x^2 - 9}{3x - 9}}$
23. $\frac{\frac{5}{x + 3}}{2 + \frac{1}{x + 3}}$
Write an equation of a parabola with vertex at the origin and the given focus.

1. focus at \((-2, 0)\)
2. focus at \((0, 4)\)
3. focus at \((0, -3)\)
4. focus at \((3, 0)\)
5. focus at \((-5, 0)\)
6. focus at \((0, 5)\)

Identify the vertex, the focus, and the directrix of the parabola with the given equation. Then sketch the graph of the parabola.

7. \(y = \frac{1}{12}x^2\)
8. \(x = -\frac{1}{4}y^2\)

9. \(y = \frac{1}{2}(x - 1)^2\)
10. \(x = -\frac{1}{4}(y + 1)^2 + 2\)

Write an equation of a parabola with vertex at the origin and the given directrix.

11. directrix \(x = 3\)
12. directrix \(y = 4\)
13. directrix \(x = -2\)
14. directrix \(y = -3\)
15. directrix \(x = 6\)
16. directrix \(y = -7\)

17. The center of a pipe with a diameter of 0.5 in. is located 10 in. from a mirror with a parabolic cross section used as a solar collector. The center of the pipe is at the focus of the parabola.

a. Write an equation to model the cross section of the mirror.
b. The pipe receives 25 times more sunlight than it would without the mirror. The amount of light collected by the mirror is directly proportional to its diameter. Find the width of the mirror.
10-3 Practice
Circles

Write an equation of a circle with the given center and radius. Check your answers.

1. center (0, 0), radius 3
2. center (0, 1), radius 2
3. center (-1, 0), radius 6
4. center (2, 0), radius 1
5. center (1, -5), radius 2.5
6. center (2, 3), diameter 1

Write an equation for each translation.
7. $x^2 + y^2 = 9$; right 4 and down 2
8. $x^2 + y^2 = 12$; left 2 and up 5
9. $x^2 + y^2 = 49$; right 1 and up 7
10. $x^2 + y^2 = 1$; right 5 and up 5
11. $x^2 + y^2 = 25$; up 10
12. $x^2 + y^2 = 36$; left 8 and down 6

Write an equation for each circle. Each interval represents one unit.

13. 

14. 

15. 

16. 

17. 

18. 

For each equation, find the center and radius of the circle.

19. $(x + 1)^2 + (y - 8)^2 = 1$
20. $x^2 + (y + 3)^2 = 9$
21. $(x + 3)^2 + (y + 1)^2 = 2$
22. $(x - 6)^2 + y^2 = 5$
23. $(x - 6)^2 + (y - 9)^2 = 4$
24. $x^2 + y^2 = 144$
10-4 Practice
Ellipses

Write an equation of an ellipse in standard form with center at the origin and with the given vertex and co-vertex listed respectively.

1. \((5, 0), (0, -5)\)
2. \((0, 10), (-7, 3)\)
3. \((0, 2), (-1, 0)\)
4. \((4, 0), (0, 2)\)
5. \((9, 0), (0, -6)\)
6. \((11, 0), (0, -10)\)
7. \((-7, 0), (0, -5)\)
8. \((-2, 0), (0, -1)\)

Find the foci for each equation of an ellipse. Then graph the ellipse.

9. \(\frac{x^2}{36} + \frac{y^2}{81} = 1\)
10. \(x^2 + \frac{y^2}{36} = 1\)
11. \(\frac{x^2}{9} + \frac{y^2}{100} = 1\)

12. \(16x^2 + 25y^2 = 1600\)
13. \(4x^2 + y^2 = 49\)
14. \(\frac{x^2}{64} + \frac{y^2}{144} = 1\)

Find the distance between the foci of an ellipse. The lengths of the major and minor axes are listed.

15. 10 and 8
16. 20 and 16
17. 30 and 16
18. 40 and 20
19. 25 and 15
20. 50 and 26

Write an equation of an ellipse for the given foci and co-vertices.

21. foci \((\pm 5, 0)\), co-vertices \((0, \pm 2)\)
22. foci \((0, \pm 2)\), co-vertices \((\pm 1, 0)\)
23. foci \((\pm 1, 0)\), co-vertices \((0, \pm 2)\)
24. foci \((0, \pm 3)\), co-vertices \((\pm 3, 0)\)
25. foci \((0, \pm 4)\), co-vertices \((\pm 4, 0)\)
26. foci \((\pm 4, 0)\), co-vertices \((0, \pm 2)\)
27. foci \((\pm 2, 0)\), co-vertices \((0, \pm 4)\)
28. foci \((\pm 1, 0)\), co-vertices \((0, \pm 5)\)